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This is a research report concerned with the availability of computer programs which made it advisable to try not only the factors dictated by a criterion, but successive additional factors until the meaning of the pattern is clarified. Examples illustrate the application of sequential factor analyses using a principal axes solution and varimax rotation. In one case, extension to an additional factor clarified the whole solution and additional factors added nothing, pointing to the suitability of a four-factor solution. In the other case, extension to one additional factor only clarified the identity of two variables and pointed back to the two factor solution as most effective. (Author/EK)

Use of Sequential Factor Analyses  
to Clarify Interpretation of Underlying Relations

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Definitive interpretation of a factor analysis is questioned by some because the constructs proposed by the investigator often appear arbitrary. An effort to meet this criticism and reduce one element of arbitrariness has been to adopt rules for extracting factors such as use of the method of principal axes for determining unrotated factor loadings, varimax rotation for maximizing zero entries to produce simple structure, and an eigenvalue criterion for how many factors to rotate.

It is the thesis of this paper that the availability of efficient computer programs makes it advisable to try not merely the number of factors dictated by a criterion, e.g., eigenvalues, but successively additional factors until the meaning of the pattern is clarified. Two examples are offered.

In the first example, we have data from the U. S. Military Academy at West Point, a highly controlled academic situation in which all students took the same 14 courses at the same year levels. Only a small number of electives (languages chiefly) were involved and grades in these subjects were omitted from the calculations. The subject titles are clear; the numbers following indicate the year in which each subject was studied (1 means senior year, 4 means freshman year). Data from two successive classes were combined, yielding 497 individuals with complete four-year records.

For purposes of this study, a principal axes solution and varimax rotation were applied. It may appear that a two-factor solution should have been used to extract a factor common to all 14 variables first, but several of the intercorrelations are below .30 and the present solution may be accepted as indicating qualitatively, if not quantitatively, the

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strength of special factors beyond a common factor of competence and effort.

In the present instance, rigorous use of an eigenvalue criterion would have dictated extracting only 3 factors. The three-factor solution, however, was difficult to interpret. Factor 1, heavily represented in Variables 1-5, is a verbal or reading factor; Factor 2, represented almost entirely in Variables 13 and 14 and somewhat in Variable 12, may be called a spatial relations factor, one that emerges in spatial relations tests that correlate highly with grades in drawing, surveying, descriptive geometry, and like subjects; Factor 3 is complex, represented in nine variables, Variables 4 - 12, but with no clear account for all these high loadings.

The four-factor solution makes great sense. Factors 1 and 2 are as before, but Factor 3 is heavily weighted in Variables 4 - 9, subjects studied in the upperclass years, while Factor 4, represented heavily in Variables 10 - 12, is a mathematical factor. Factor 3 is explained as arising from a tendency in upperclass years to have established one's strengths and weaknesses and to be concentrating on keeping one's head equally above water in all subjects.

The five-factor solution results in essentially the same first four factors, with the fifth raising the communality of Variable 12 from lowest to highest in one stroke. The six-factor solution adds a sixth factor which raises the communality for Variable 3 from lowest to highest.

The four-factor solution seems best, further factors seeming only to "scavenge" for unaccounted variance in an unreliable fashion.

In the second example, the intercorrelations are among judgments of agreement or disagreement on twelve clusters of policy statements, each represented by a summary statement. The policy statements were taken from the published positions of the American Personnel and Guidance Association in the general area of individual rights. A total of 485 had responded to a questionnaire on these items.

Again, a principle axes solution and varimax rotation were used and two factors were indicated by the eigenvalue criterion. The two-factor solution was clear. Factor 1, represented substantially in Variables 1 - 6, 11 - 12 reflects affirmation of the basic counselor-counselee relationship of support in making sound personal decisions. Factor 2,

represented substantially in Variables 7 - 10, is one of responsibility to the public or society. This second factor is a secondary characteristic in responses to statements 11 and 12, while the first factor plays a secondary role in responses to statements 7 and 9.

The three-factor solution yields evidence that responses to statements 4 and 5 are so highly correlated as to make the two appear largely the same. The correlation of .52 exceeds by .12 the nearest other values in the original table. Having established this, it appears best to return to the two-factor solution for general interpretations. Only statement 7 presents a problem of classification and this is simply recognized to reflect the predominance of the public responsibility factor in the final phrase "what he may become."

In general, we have illustrated by two examples application of sequential factor analyses to clarify the factors operating in a field situation. In one case, extension to one additional factor clarified the whole solution and additional factors added nothing. In the other case, extension to one additional factor only clarified the identity of two variables and pointed back to the two-factor solution as most efficient.

AERA

Los Angeles

February 6, 1969



Brigham, Carl C., A Study of Error New York: College Entrance Examination Board, 1932.

Correlation Matrix, p. 38 (West Point Grades) (N=487)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1. Engl. 3		.77	.68	.62	.59	.47	.46	.47	.50	.43	.41	.39	.28	.26
2. Engl. 4	.77		.61	.56	.57	.41	.38	.42	.45	.43	.50	.36	.20	.19
3. Hist. 3	.68	.61		.69	.60	.53	.51	.52	.53	.45	.39	.38	.24	.23
4. Econ. 1	.62	.56	.69		.82	.73	.72	.66	.70	.53	.46	.49	.27	.25
5. Law 1	.59	.57	.60	.82		.73	.73	.69	.73	.56	.48	.51	.22	.19
6. Engg. 1	.47	.41	.53	.73	.73		.92	.88	.85	.75	.60	.67	.54	.51
7. Ordn. 1	.46	.38	.51	.72	.73	.92		.87	.84	.72	.56	.64	.54	.51
8. Phys. 2	.47	.42	.52	.66	.69	.88	.87		.91	.82	.66	.71	.57	.54
9. Chem. 2	.50	.45	.53	.70	.73	.85	.84	.91		.78	.66	.72	.51	.47
10. Math 3	.43	.43	.45	.53	.56	.75	.72	.82	.73		.82	.68	.51	.50
11. Math 4	.41	.50	.39	.46	.48	.60	.56	.66	.66	.82		.68	.39	.39
12. Surv. 4	.39	.36	.38	.49	.51	.67	.64	.71	.72	.68	.68		.56	.57
13. Dng. 2	.28	.20	.24	.27	.22	.54	.54	.57	.51	.51	.39	.56		.92
14. Dng. 3	.26	.19	.23	.25	.19	.51	.51	.54	.47	.50	.39	.57	.92	

Three Factor Solution

Test	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	h <sup>2</sup>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	h <sup>2</sup>
1	.88	.18	.22	.85	.87	.15	.23	.16	.85
2	.87	.13	.19	.80	.86	.04	.11	.32	.86
3	.76	.08	.35	.71	.73	.09	.43	.07	.73
4	.59	.02	.66	.79	.53	.04	.75	.11	.85
5	.53	.08	.73	.81	.46	-.05	.76	.21	.84
6	.26	.28	.65	.87	.19	.32	.80	.35	.90
7	.24	.28	.85	.85	.18	.33	.82	.30	.90
8	.24	.34	.85	.90	.19	.34	.72	.47	.90
9	.29	.27	.85	.88	.23	.26	.73	.47	.88
10	.21	.37	.76	.76	.18	.28	.46	.74	.87
11	.26	.32	.63	.57	.25	.16	.24	.88	.91
12	.18	.48	.65	.68	.15	.41	.40	.61	.73
13	.08	.91	.28	.91	.08	.93	.21	.18	.95
14	.07	.92	.25	.92	.08	.94	.17	.19	.95

Five Factor Solution

Test	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	h <sup>2</sup>	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	F <sub>4</sub>	F <sub>5</sub>	F <sub>6</sub>	h <sup>2</sup>
1	.87	.16	.23	.15	.02	.85	.83	.16	.27	.10	.05	.24	.87
2	.86	.05	.12	.34	-.03	.87	.90	.05	.18	.25	.03	.09	.92
3	.74	.09	.41	.03	.16	.74	.46	.08	.35	.13	.05	.79	.98
4	.54	.02	.73	.03	.21	.87	.42	.03	.74	.05	.19	.34	.87
5	.47	-.06	.75	.15	.15	.84	.44	-.05	.79	.11	.18	.11	.88
6	.20	.31	.83	.29	.08	.91	.15	.32	.82	.31	.08	.13	.91
7	.19	.32	.85	.24	.06	.91	.13	.33	.84	.26	.06	.11	.91
8	.19	.33	.76	.43	.07	.91	.15	.34	.74	.45	.07	.12	.91
9	.24	.25	.75	.41	.16	.88	.20	.26	.75	.42	.17	.10	.88
10	.18	.28	.52	.71	.03	.90	.15	.28	.49	.75	.02	.13	.93
11	.24	.16	.29	.84	.19	.91	.26	.15	.27	.84	.21	.07	.92
12	.15	.38	.39	.46	.66	.97	.14	.38	.39	.43	.69	.06	.98
13	.08	.93	.24	.15	.06	.95	.07	.93	.22	.16	.07	.04	.95
14	.08	.93	.19	.16	.10	.95	.07	.94	.18	.16	.11	.05	.95

Six Factor Solution

# Example #2

## Intercorrelation of Responses to 12 APGA Questionnaire Items

Item	1	2	3	4	5	6	7	8	9	10	11	12
1	--	.31	.35	.31	.30	.33	.22	.07	.29	.09	.29	.22
2	.31	--	.32	.30	.25	.26	.40	.28	.30	.18	.31	.26
3	.35	.32	--	.31	.22	.35	.26	.17	.21	.09	.28	.31
4	.31	.30	.31	--	.52	.34	.21	.33	.25	.18	.32	.32
5	.30	.25	.22	.52	--	.35	.23	.27	.32	.18	.28	.25
6	.33	.26	.35	.34	.35	--	.29	.13	.26	.11	.38	.30
7	.22	.40	.26	.21	.23	.29	--	.34	.33	.17	.32	.33
8	.07	.28	.17	.33	.27	.13	.34	--	.40	.32	.30	.29
9	.29	.30	.21	.25	.32	.26	.33	.40	--	.21	.30	.32
10	.09	.18	.09	.18	.18	.11	.17	.32	.21	--	.21	.19
11	.29	.31	.28	.32	.28	.38	.32	.30	.30	.21	--	.23
12	.22	.26	.31	.32	.25	.30	.33	.29	.32	.19	.29	--

Two-Factor Solution				Three-Factor Solution				
Item	F <sub>1</sub>	F <sub>2</sub>	h <sup>2</sup>	Item	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>	h <sup>2</sup>
1	.71	-.02	.51	1	.58	-.13	.39	.51
2	.47	.38	.37	2	.62	.29	.05	.47
3	.66	.07	.44	3	.68	-.04	.20	.50
4	.58	.31	.43	4	.20	.24	.76	.68
5	.54	.30	.38	5	.13	.24	.79	.70
6	.70	.09	.50	6	.55	-.02	.45	.50
7	.36	.50	.38	7	.62	.43	-.11	.58
8	.07	.81	.65	8	.13	.79	.14	.66
9	.32	.57	.43	9	.37	.51	.18	.43
10	-.04	.66	.43	10	-.03	.66	.14	.46
11	.49	.39	.39	11	.48	.31	.27	.39
12	.42	.42	.36	12	.47	.35	.17	.37

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### Policy Statements

- \_\_\_\_\_ 1. The counselor truly accepts and believes in the worth and potential of each and every individual he counsels.
- \_\_\_\_\_ 2. The counselor recognizes and accepts the counselee's need for self-understanding as a basis for making his own decisions.
- \_\_\_\_\_ 3. The counselor respects the individuality of each and every counselee by letting him work out his own values and set his own goals within a broad, but not unlimited range, providing assistance as needed.
- \_\_\_\_\_ 4. The counselor actively seeks to widen opportunities for his counselees to gain useful experience in making decisions by working with other staff personnel to remove unnecessary restrictions.
- \_\_\_\_\_ 5. The counselor assists other staff members in actively making adaptations to individual differences among counselees.
- \_\_\_\_\_ 6. The counselor resists pressures that would restrict his counselee's rights, including his self-respect.
- \_\_\_\_\_ 7. The counselee should be helped to understand and accept himself not only for what he is, but what he may become.
- \_\_\_\_\_ 8. Occupational information is basic to sound vocational planning.
- \_\_\_\_\_ 9. Maximum development of individuals is to be sought for the good of the individual and society.
- \_\_\_\_\_ 10. The counselee's parents should be informed of his status and progress so as to further their responsibility for maximizing his development.
- \_\_\_\_\_ 11. Use of materials or data generated in the counseling relationship for secondary purposes must safeguard the primary objective, which is to promote the counselee's best interests.
- \_\_\_\_\_ 12. It is in the public interest to require broad educational backgrounds in counselors, and training programs that are adaptive to their individual differences and needs.